Určete přibližně

$$\ln\left(\sqrt{9{,}03} - \sqrt{0{,}99} - 1\right)$$

- $\ln\left(\sqrt{9} - \sqrt{1} - 1\right) = \ln 1 = 0$

$9 \rightsquigarrow 9{,}03 \quad , \quad 1 \rightsquigarrow 0{,}99$

2 různé kroky $\Rightarrow$ musíme dát 2 prom.

$$f(x,y) = \ln\left(\sqrt{x} - \sqrt{y} - 1\right)$$

Chceme aproximaci v okolí bodu $(9,1) =: a$

Chceme $df(a)$ ... aproximuje přírůstky fce f:

$$f(a+h) - f(a) = df(a)(h) - \eta(h)$$

$$f(a) = f(9,1) = \ln 1 = 0$$

$$h = (0{,}03,\ -0{,}01)^T \in \mathbb{R}^2$$

Chceme $df(a)$ ... $\left(\frac{\partial f}{\partial x}(a),\ \frac{\partial f}{\partial y}(a)\right)$

- $\frac{\partial f}{\partial x}(x,y) = \frac{\partial}{\partial x}\left(\ln\left(\sqrt{x} \underbrace{- \sqrt{y} - 1}_{\text{konst.}}\right)\right) =$

$$= \frac{1}{\sqrt{x} - \sqrt{y} - 1} \cdot \frac{+1}{2\sqrt{x}}$$

- $\frac{\partial f}{\partial y}(x,y) = \frac{1}{\sqrt{x} - \sqrt{y} - 1} \cdot \frac{-1}{2\sqrt{y}}$

$$df(a) = df(9,1) = \left(\frac{1}{\sqrt{9} - \sqrt{1} - 1} \cdot \frac{+1}{2\sqrt{9}},\ \frac{1}{\sqrt{9} - \sqrt{1} - 1} \cdot \frac{-1}{2\sqrt{1}}\right)$$

$$= \left(\frac{+1}{6},\ \frac{-1}{2}\right)$$

$$f(9{,}03,\ 0{,}99) = f(a+h) \cong f(a) + df(a)(h) =$$

$$= 0 + \left(+\frac{1}{6}\ \ -\frac{1}{2}\right)\begin{pmatrix} 0{,}03 \\ -0{,}01 \end{pmatrix} = \frac{+0{,}03}{6} + \frac{0{,}01}{2} = +\frac{1}{100}$$

Tečná rovina: v bodě $(a,b) \in \mathbb{R}^2$

$$\boxed{z} = f(a,b) + \frac{\partial f}{\partial x}(a,b)\cdot(\boxed{x}-a) + \frac{\partial f}{\partial y}(a,b)\cdot(\boxed{y}-b)$$

$$= f(a,b) + df(a,b)\big(\underbrace{(x-a, y-b)}_{h}\big)$$

4d): $f(x,y) = x^2 \cdot \cos\frac{1}{y}$ $\qquad A = [1, \frac{2}{\pi}, ?]$

$$? = f(1, \tfrac{2}{\pi}) = 1^2 \cdot \cos\frac{\pi}{2} = 0$$

$$A = [1, \tfrac{2}{\pi}, 0]$$

Tečná rovina ke grafu f v bodě A

$$A = (1, \tfrac{2}{\pi}, f(1, \tfrac{2}{\pi})).$$

$$\frac{\partial f}{\partial x}(x,y) = 2x \cdot \cos\frac{1}{y}$$

$$\frac{\partial f}{\partial y}(x,y) = x^2 \cdot \left(-\sin\frac{1}{y}\right)\cdot\frac{-1}{y^2} = \frac{x^2}{y^2}\cdot\sin\frac{1}{y}$$

$$\frac{\partial f}{\partial x}(1, \tfrac{2}{\pi}) = 2\cdot 1\cdot\cos\frac{\pi}{2} = 0$$

$$\frac{\partial f}{\partial y}(1, \tfrac{2}{\pi}) = \frac{1^2}{(\frac{2}{\pi})^2}\cdot\sin\frac{\pi}{2} = \frac{\pi^2}{4}$$

Rovnice tečné roviny:

$$z = 0 + 0\cdot(x-1) + \frac{\pi^2}{4}\left(y - \frac{2}{\pi}\right) =$$

$$z = -\frac{\pi}{2} + \frac{\pi^2 y}{4}$$ rovnice tečné roviny

5a) Tečnou rovinu ke grafu f rovnoběžnou s ρ:

$$f(x,y) = x^2 + xy - y^2 + x + 3$$

$$\rho: \; 5x - 3y - z = 0$$

$$\rho: \; z = 5x - 3y \quad \Big| \quad \left(\frac{\partial f}{\partial x}(a,b), \frac{\partial f}{\partial y}(a,b)\right) = t\cdot(5,-3)$$

$$y^{x^2+3} = e^{\ln\left(y^{x^2+3}\right)} = e^{(x^2+3)\cdot \ln y}$$

$$\frac{d}{dx}\left(y^{x}\right) = \left(e^{x\ln y}\right) = \ln y \, e^{x \ln y} = \ln y \cdot y^{x}$$

$$(a^{x})' = \ln a \cdot a^{x}$$

$$f(x,y) = (2x-3y)^4$$

$$\frac{\partial f}{\partial x}(x,y) = 4(2x-3y)^3 \cdot 2$$

$$\frac{\partial f}{\partial y}(x,y) = 4(2x-3y)^3 \cdot (-3)$$

b) y^{x^2+3}

$$\frac{\partial}{\partial x}\left(y^{x^2+3}\right) = \frac{\partial}{\partial x}\left(e^{(x^2+3)\ln y}\right) =$$

$$= e^{(x^2+3)\ln y} \cdot \ln y \cdot 2x$$

$$\frac{\partial}{\partial y}\left(y^{x^2+3}\right) = (x^2+3)\cdot y^{x^2+2}$$

d) $f(x,y,z) = (x^{y})^{z} = x^{yz}$

- $\frac{\partial f}{\partial x}(x,y,z) = yz \cdot x^{yz-1}$ | pokud $yz \neq 0$

Pokud $yz = 0 \Rightarrow \frac{\partial f}{\partial x} = 0$.

- $\frac{\partial f}{\partial y} = ?$ $\quad x^{yz} = e^{\ln\left(x^{yz}\right)} = e^{yz \cdot \ln x}$

$$\frac{\partial f}{\partial y} = z\cdot(x^y)^{z-1}\cdot \ln x \cdot x^y$$

$$f(x,y,z) = (x^y)^z = \left(e^{y\ln x}\right)^z =$$

$$= e^{z\ln e^{y\ln x}} = e^{zy\ln x} = x^{zy}$$

$$\frac{\partial f}{\partial y}(x,y,z) = \frac{\partial}{\partial y}\left(e^{zy\ln x}\right) =$$

$$= e^{zy\ln x}\cdot z\ln x = (x^y)^z\cdot z\ln x \checkmark$$

$$\frac{\partial f}{\partial z}(x,y,z) = (x^y)^z\cdot y\ln x \checkmark$$

$$\frac{\partial f}{\partial x}(x,y,z) = \frac{\partial}{\partial x}\left(x^{zy}\right) = zy\cdot x^{zy-1}. \checkmark$$

5a) $\frac{\partial f}{\partial x} = 2x+y+1$

$\frac{\partial f}{\partial y} = -2y+x$

Tečná rovina v bodě $(a,b,?)$

$$z = f(a,b) + (2a+b+1)(x-a) + (-2b+a)(y-b)$$

$\boxed{(a,b)=\frac{1}{2}}$ $\quad z = f(1,2) + 5\cdot(x-1) + (-3)(y-b)$

$\rho: \quad z = 5x - 3y$

$$a = \frac{7}{5t} - \frac{2}{5}$$

$$b = \frac{11}{5t} - \frac{1}{5}$$

$$t(2a+b+1) = 5$$

$$t(a-2b) = -3$$

$\underline{t=1}$ $\quad \boxed{\begin{matrix} a=1 \\ b=2 \end{matrix}}$

$$2a+b = \frac{5}{t} - 1$$

$$a-2b = \frac{-3}{t}$$

$$5b = \frac{5}{t} - 1 + \frac{6}{t} \quad \Big| \quad 5a = \frac{7}{t} - 2$$